# A stochastic study of the hygro-elastic behavior of composite materials: application to the durability of renewable marine energy structures

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**Abstract**

*The present paper investigates the modelling of the hygro-mechanical behaviour of composited used in renewable marine energy structures. The objective is to analyze the effect of moisture diffusion on a composite structure and its mechanical states. We especially focus on the impact of cracks on both diffusion and elastic quantities of interest. We base our study on a classical Fick diffusion model coupled with linear elastic constitutive equations allowing studying the hygroscopic swelling. Furthermore we address the tackle of uncertainties usually observed in this type of problem. The so-called stochastic hygro-elastic problem is based on a polynomial chaos approximation and solved with a non-intrusive technique allowing using the classical finite element method to evaluate the solution for particular realizations of the random variables. We finally propose to illustrate the proposed methodology with a numerical application at the microscale involving a composite sample with a crack.*

*Keywords: composite materials; ageing; transient hygro-mechanical problem; multi-physics; uncertainties.*

## 1. Introduction

Nowadays, composite materials are often used in structural design for various engineering applications thanks to their good mechanical properties coupled to their lightness. Those materials are presently more and more used in renewable marine energy structures for the manufacturing of structural parts such as offshore windmill blades. During their life-service, these components are submitted to harsh environments which, combined to classic mechanical loadings, may lead to a premature aging of the structure [1]. Among these various aggressive phenomena, water absorption is of first importance since the involving hygroscopic swelling may activate or worsen a damage mechanism such as crack initiation or propagation. In this work, we thus focus on the impact of water absorption on structural parts made of composites the polymer matrix of which can be hydrophilic. The diffusion model is

the classical Fick's law [2] and the mechanical problem is solved under a linear elasticity assumption. Many experimental and numerical studies have been devoted to moisture diffusion in composite materials and its impact on the mechanical behavior such as [3, 4, 1, 5, 6].
Experiments conducted in order to quantify the diffusion parameters (diffusion tensor and maximum moisture absorption capacity) show a quite large dispersion observed on the identified diffusion parameters. A stochastic study seems necessary in order to well apprehend the uncertainties on the various output fields such as local water content or stress fields induced by the so-called hygroscopic swelling. Here is the purpose of the present work.
We adopt a parametric vision of the uncertainties which leads to a probabilistic model based on independent random variables [7]. These random variables help in the modeling of parameters such as the water diffusion coefficient or the maximum moisture absorption capacity. We focus on the propagation of uncertainties through the proposed physical model governed by stochastic partial differential equations. Several methods are available to achieve this task depending on the probabilistic quantities one seeks to obtain (Monte-Carlo methods, reliability analysis etc.). Among them, spectral stochastic methods are good candidates in order to get an explicit solution with respect to the basic random variables modeling the diffusion coefficients, for instance. They consist in representing the random solution on a suitable approximation basis.
The last section of the present paper is dedicated to the study of a slice of a composite sample containing an edge crack from which moisture can penetrate within the material. The length of the crack is assumed random as the maximum moisture content. The proposed numerical method is applied to quantify the impact of those uncertainties on the hygro-mechanical response of the composite.

## 2. Fickean hygro-mechanical deterministic problem

In this section, we present the diffusion model used in this work. We focus on the classical Fick model allowing representing classical sorption phenomena [2]. We also present the mechanical constitutive equations which take into account the hygroscopic swelling and help complete the hygro-elastic model.

We consider a heterogeneous material, as the one depicted on figure 2 at the beginning of the numerical study, which occupies a spatial domain. $\Omega_m$ and $\Omega_r$ respectively represent the matrix and the fibers acting as reinforcements. The variable $c(\mathbf{x},t)$ denotes the moisture content of a material point, characterized by its position through vector $\mathbf{x}$, at time $t$. This moisture content $c(\mathbf{x},t)$ is defined by

$$c(\mathbf{x},t) = \frac{m_w(\mathbf{x},t)}{m_0(\mathbf{x})}, \tag{1}$$

where $m_w(\mathbf{x},t)$ is the local uptake in mass of water whereas $m_0(\mathbf{x})$ is the local mass at the initial time.
The spatial average moisture content $C(t)$ can be obtained with the following relation

$$C(t) = \frac{1}{M_0} \int_{\Omega} \rho(\mathbf{x}) c(\mathbf{x},t) d\Omega, \tag{2}$$

where $\rho(\mathbf{x})$ is the local density and $M_0$ is the mass of the sample at initial time. From an experimental point of view, using the knowledge on $M_0$ and the mass $M(t)$ at time $t$ of the sample, the overall volume of which is $\Omega$, $C(t)$ is evaluated using the following relationship

$$C(t) = \frac{M(t) - M_0}{M_0}. \tag{3}$$

Gravimetric measurements through time thus allow the characterization of the water diffusion kinetics. From now on, all equations will be written according to the local moisture content $c(\mathbf{x},t)$. In this work, we assume that the diffusion process is governed by a unique diffusion coefficient $D$ in each spatial direction. Moreover, since the fibers are considered hydrophobic, the problem may be solely formulated on the domain $\Omega_m$.

The Fick law is a classical choice to represent a diffusion process where each water molecule is free to move in the polymer network associated with domain $\Omega_m$. Since the diffusivity is assumed constant (*i.e.* independent of moisture content or mechanical states), the Fick local diffusion problem writes: find the solution field $c(\mathbf{x},t)$ such that it verifies

$$\begin{array}{rcll} \dfrac{\partial c}{\partial t} & = & D\Delta c & on\ \Omega_m, \\ c & = & c^{\infty} & on\ \Gamma_c, \end{array} \tag{4}$$

where $c^{\infty}$ is a given moisture content applied on a part $\Gamma_c$ of boundary $\partial\Omega$ which represents the maximum moisture content. One should note that other diffusion models can be used in order to repesent diffusion kientics exhbiting soprtion anomalies. Among them we find the Langmuir model [8, 5] or the dual stage Fick model [9].

The previous diffusion problem can be coupled to an elastic problem in order to analyze the deformation of the material or the structure under aging conditions. We denote by $\mathbf{u}(\mathbf{x},t)$ the displacement field, by $\mathbf{Ů}(\mathbf{u}(\mathbf{x},t))$ the strain tensor, and by $\mathbf{ů}(\mathbf{x},t)$ the Cauchy stress tensor. Both fibers and matrix are assumed to be linear isotropic elastic materials represented by the fourth order stiffness tensor $\mathbf{C}$ verifying

$$\mathbf{C}(\mathbf{x}) = \begin{cases} \mathbf{C}_m & \text{if } \mathbf{x} \in \Omega_m \\ \mathbf{C}_r & \text{if } \mathbf{x} \in \Omega_r \end{cases}, \tag{5}$$

where $\mathbf{C}_m$ and $\mathbf{C}_r$ are constant tensors. We also introduce $\beta_h$ the hygroscopic expansion coefficient which is considered, in this work, identical in each direction. The hygroscopic expansion is represented by the diagonal tensor $\mathbf{ƃ}_h$ whose diagonal components are equal to $\beta_h$. Material parameter $\beta_h$ is taken equal to $0$ for the hydrophobic reinforcements. Finally, the quasi-static linear elastic problem writes: find the displacement field $\mathbf{u}(\mathbf{x},t)$ such that

$$
\begin{array}{rcll}
\nabla \cdot \boldsymbol{\sigma} & = & \mathbf{0} & \textit{on}\ \Omega, \\
\boldsymbol{\sigma} & = & \mathbf{C} : \left[\boldsymbol{\varepsilon} - \boldsymbol{\beta}_h c\right] & \textit{on}\ \Omega, \\
\mathbf{u} & = & \mathbf{u}_{imp} & \textit{on}\ \Gamma_u,
\end{array}
\tag{6}
$$

where $\mathbf{u}_{imp}$ is the imposed displacement on the part $\Gamma_u$ of $\partial\Omega$ and $c(\mathbf{x},t)$ is the solution field of problem (4).
In this work, simulations involving Fick or Langmuirôs models were performed using commercial finite element software Abaqus in which a specific element were developed using a UEL subroutine. The corresponding elastic problems have also been solved using the same finite element software. In this work, we do not introduce dependencies between the diffusion or elastic properties and the local fields such as the moisture content or the strains. However, this could be done using coupled models such as the ones proposed in [10, 11]
The next section is devoted to tackle uncertainties of different nature which can be usually observed for the hygro-elastic study of composites.

## 3. Stochastic hygro-mechanical problem

The present section is dedicated to take into consideration the uncertainties which may appear in a hygro-elastic problem. Figure 1 presents the moisture uptake through time of composite samples. Even if all samples are assumed to be the same (*i.e.* same material and same geometry) we can observe relevant differences for both coefficient of diffusion and maximum water content. Taking into account solely the mean value or extreme values of the diffusion parameters may lead to quite poor predictions. Probabilistic approaches thus aim at considering the uncertainties observed on the input data and at propagating them in the physical system. The response thus becomes random and allows most robust and reliable predictions. We propose a methodology to efficiently propagate the uncertainties based on a parametric vision and polynomial chaos approximation.
We are now interested in taking into account the uncertainties on the diffusive properties of the material. For the sake of simplicity, we solely present the stochastic problem for the diffusion problem (4) corresponding to the Fick model. To achieve this task, we adopt a parametric vision of the uncertainties resulting in working in a probability finite dimension space. The probabilistic content is then represented by a set of independent random variables $\boldsymbol{\xi} = (\xi_1, \ldots, \xi_m)$ with probability law $P_{\boldsymbol{\xi}}$. We introduce the associated probability space $(\Theta, B_\Theta, P_\xi)$, where $\Theta \subset \mathbb{R}^m$ is the set of elementary events and $B_\Theta$ is a $\sigma$-algebra on $\Theta$. Random variables $\boldsymbol{\xi}$ are used to characterize the various random inputs, such as the material properties or loadings.

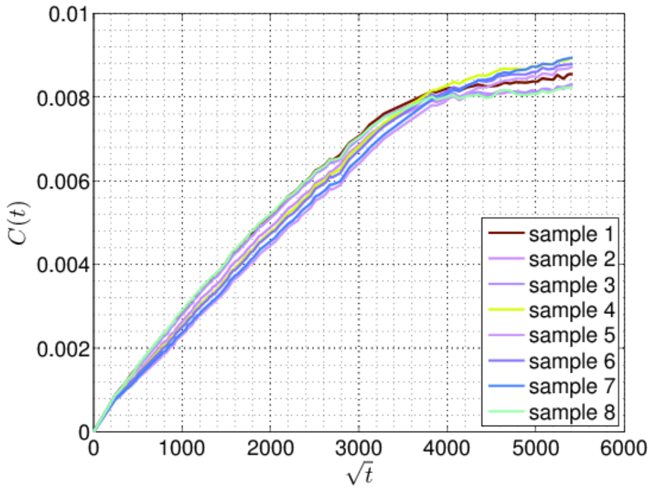



**Figure 1.** Experimental sorption curves for a unidirectional composite exhibiting the uncertainties of the process.

The introduction of uncertainties modifies problem (4) which becomes: find local moisture field $c(\mathbf{x},t,\xi):\Omega_m \times \mathsf{T} \times \bar{\mathbf{U}}$ such that

$$\begin{aligned} \frac{\partial c(\mathbf{x},t,\xi)}{\partial t} &= D(\xi)\Delta c(\mathbf{x},t,\xi) \quad on\ \Omega_m \times T \times \bar{\mathbf{U}}, \\ c(\mathbf{x},\xi) &= c^{\infty}(\mathbf{x},\xi) \qquad on\ \Gamma_c \times \bar{\mathbf{U}}, \end{aligned} \tag{7}$$

In order to quantify the random local response $c(\mathbf{x},t,\xi)$, it is necessary to use dedicated computational techniques. The most famous of them is the Monte Carlo method which consists in generating a large number of realizations of $\xi$ and to evaluate the system response $c(\mathbf{x},t,\xi)$ for each of them [7, 12]. This non-intrusive technique is easy to implement and especially provides a good estimate of the statistical moments. However, it may require a large number of deterministic calculations and leads to significant computational times. We then choose to use a different calculation technique, belonging to spectral stochastic methods, and leading to a functional representation of the random response, which requires a few deterministic calculations when the number of random input variables $m$ is low enough.

As it has been done for the spatial-time problem for which we introduced a spatial approximation space thanks to finite element, it is necessary to define a stochastic approximation space $\mathsf{S}_P \subset \mathsf{S} = L^2(\bar{\mathbf{U}}, dP_\xi)$ where $\mathsf{S}$ is the second-order random variable space. The stochastic approximation space $\mathsf{S}_P$ can always be written

$$\mathsf{S}_P = \{v(\xi) = \sum_{\alpha=1}^{P} v_\alpha H_\alpha(\xi),\ v_\alpha \in \mathsf{R}\}, \tag{8}$$

where $\{H_\alpha\}_{\alpha=1}^{P}$ is a functional basis of $\mathsf{S}$ for which several choices exist such as polynomial chaos expansion and its generalization [13, 14]. These types of representation are well adapted for regular functions with respect to input random

variables. When those functions are less regular (for instance a continuous function whose derivatives are discontinuous) a high number $P$ of basis functions may be needed. In this case, other ways of representation may be used such as piecewise polynomial approximation [15] or multi-wavelets [16].
In order to solve the problem (7) we choose to use a $L^2$ projection method at the stochastic level [17] which only requires the resolution of problem (4) for a final set of realizations of basic random variables.

The aim is to seek a functional form of the semi-discretized solution $\mathbf{c}(t_n, \boldsymbol{\xi}) \in \mathbb{R}^Q \, \forall 0 \leq t_n \leq T$ where $Q = N \times P$ ans such that

$$\mathbf{c}(t_n, \boldsymbol{\xi}) \approx \sum_{\alpha=1}^{P} \mathbf{c}_\alpha(t_n) H_\alpha(\boldsymbol{\xi}) \tag{9}$$

The $L^2$ projection technique consists in defining approximation (9) as the projection of $\mathbf{c}(t_n, \boldsymbol{\xi})$ onto the subspace $\mathcal{S}$ spanned by the basis functions $\{H_\alpha\}_{\alpha=1}^{P}$. This projection is defined with respect to the usual inner product of $\mathcal{S}$

$$\langle v, w \rangle_{\mathcal{S}} = E(v(\boldsymbol{\xi}) w(\boldsymbol{\xi}))) = \int_{\Xi} v(\mathbf{y}) w(\mathbf{y}) dP_\xi(\mathbf{y}). \tag{10}$$

When considering orthonormal functions $H_\alpha$, the coefficients $\mathbf{c}_\alpha(t_n) \in \mathbb{R}^N$ of the approximate solution (15) are then defined by

$$\mathbf{c}_\alpha(t_n) = E(\mathbf{c}(t_n, \boldsymbol{\xi}) H_\alpha(\boldsymbol{\xi})), \tag{11}$$

where $E(\cdot)$ denotes the mathematical expectation. Computing coefficients $\mathbf{c}_\alpha(t_n)$, and thus the discretized solution, leads to approximate the expectation from below. This task can be achieved with a suitable numerical integration at the stochastic level (quadrature, Monte-Carlo, etc.) which writes

$$\mathbf{c}_\alpha(t_n) = \sum_{k=1}^{K} \omega_k \mathbf{c}(t_n, \boldsymbol{\xi}_k) H_\alpha(\boldsymbol{\xi}_k), \tag{12}$$

where the $\boldsymbol{\xi}_k$ and $\omega_k$ are respectively the integration points ans the associated integration weights. These points correspond to particular realizations of input random variables $\boldsymbol{\xi}$. This approach is thus clearly a non-intrusive technique since is only requires the resolution of $K$ deterministic problems, for instance, with the use of a simple deterministic finite element solver. On the next section, we propose a numerical study showing the capabilities of the proposed stochastic approach applied to the hygro-mechanical behavior of a unidirectionnal composite material containing a crack.

## 4. Numerical study

This last section is dedicated to a numerical study involving a composite material submitted to uncertainties. The problem is illustrated on figure 2. We focus on a 2D

representation of a slice of a sample leading to the problem of a plate of size $75.\times 150\ \mu m^2$. This sample contains an edge crack from which moisture can penetrate within the material. The objective is to analyze the impact of the length of this crack on both the diffusion behavior of the composite and the internal stresses involved by the hygroscopic swelling. Moreover, the length of this crack is assumed random.
The probabilistic content is represented by two independent uniform random variables which model the length of the edge crack $L_{crack}$ located at the center of the specimen, and the maximum moisture content $c^{\infty}$ such that : $c^{\infty} \in \mathrm{U}(1.5, 1.9)\ \ [\%H20]$ and $L_{crack} \in \mathrm{U}(7.1, 36)\ \ [\mu m]$. All the other parameters are assumed deterministic. The volume fraction of the fibers is equal to 40% and their diameter is equal to 17 $\mu m$. The diffusion parameter is equal to $82\,10^{-3}\,\mu m^2$. The maximum moisture content $c^{\infty}$ is used as a boundary condition on the whole boundary of the domain and along the crack as shown in blue on figure 2.

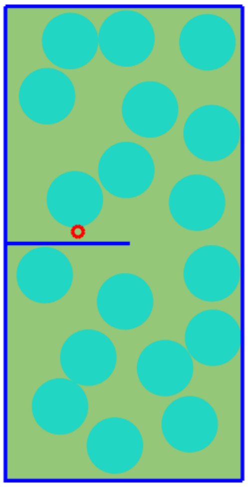
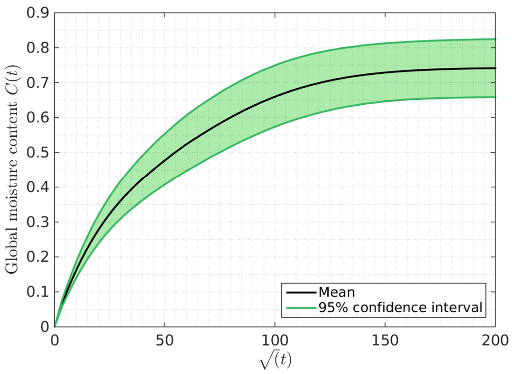


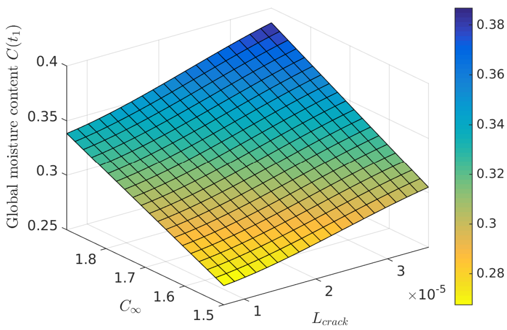
**Figure 2.** Presentation of the hygro-mechanical problem.

The elastic parameters are also all deterministic with Young modulus $E = 4\ GPa$, Poisson ratio $\nu = 0.36$ and isotropic hygroscopic expansion coefficient $\beta = 0.00324$ for the matrix. For the fibers, the elastic properties are : $E = 72\ GPa$, $\nu = 0.22$ and $\beta = 0$. The initial water content is assumed equal to zero within the sample. In the elsatic analysis, the hygroscopic swelling involved by the moisture content is taken into account without any other external or internal forces. For the spatial approximation we use a finite element mesh composed of around 27000 3-nodes linear triangle elements. For the time approximation, an implicit Euler forward scheme is used with a time step increment of 100 seconds and a total time close to 11 hours. At the stochastic level, we use a polynomial chaos with degree $p = 3$ and basis function $\{H_{\alpha}\}$ are the Legendre polynomials corresponding to uniform distribution. This polynomial order allows representing the possible nonlinear behavior of the stochastic quantities of interest depicted in the following. One should note that both spatial, time and stochastic convergence have been checked to provide accurate results in the following.

We first focus on the so-called sorption curve representing the evolution of the global moisture content $C(t)$ depicted on Figure 3-a. First, we can clearly observed a Fickean diffusion behavior as expected. We can also remark that, even if the diffusion coefficient is deterministic, the transient part of the process is submitted to relevant randomness: this can be explained by the uncertainties on the maximum moisture content and on the length of the crack which can accelerate the diffusion process depending on its size. The uncertainties observed when the saturation state is reached only come from the randomness on the maximum moisture content. Figure 3-b illustrates the response surface of the same global moisture content for a particular time instant belonging to the transient part: we can observe that both random parametric parameters have an influenced on this quantity of interest. We can also notice a linear behavior with respect to both parameters.



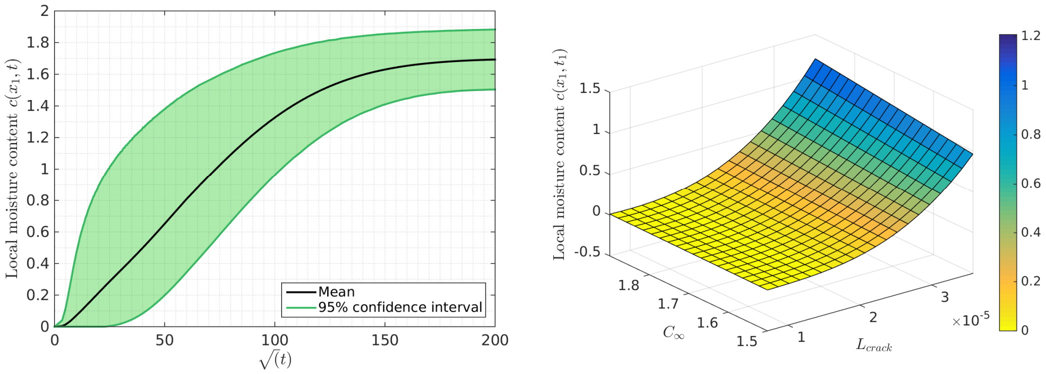

(a)




(b)

**Figure 3.** Evolution of the global moisture content $C(t)$ [%H20] versus time $[\sqrt{s}]$ (a) and response surface of $C(t_1)$ with respect to $L_{crack}$ and $c^{\infty}$ (b) .

We then focus on a local diffusion quantity. Figure 4-a represents the evolution of the local moisture content for a particular material point near the edge crack depicted on figure 2 with a red circle. We can observe very important uncertainties on this quantity based on the 95% confidence interval, especially for the transient part of the process: this result can be explained by the location of this point which can be near or far from the crack depending on its length. When the length of the crack is high, the diffusion starts very quickly, but, when its length is low, a delay of the diffusion process occurs. Figure 4-b illustrates the response surface of this local moisture content for a particular time belonging to the transient part. We can notice that only the length of the crack has a significant influence on this quantity. Moreover, its behaviour is nonlinear according to the random parameter $L_{crack}$ . The influence of the uncertainties on the maximum moisture content $c^{\infty}$ is only remarkable at the saturated state of the process.




(a) (b)

**Figure 4.** Evolution of the local moisture content $c(\mathbf{x}_1, t)$ [%H20] versus time $[\sqrt{s}]$ (a) and response surface of $c(\mathbf{x}_1, t_1)$ with respect to $L_{crack}$ and $c^{\infty}$ (b) .

We finally propose to show some results regarding the elastic behaviour of the composite involved by the hygroscopic swelling. Figure 4 presents the stress field $\sigma_{yy}$ at the saturated state for particular realizations of the random variables corresponding to their maximum values. We can observe relevant internal stresses only due to the hygroscopic swelling. Those important levels in tensile state occurs when two fibers are very close to each other and indicate, for instance, a possible debonding between the fiber and the matrix.

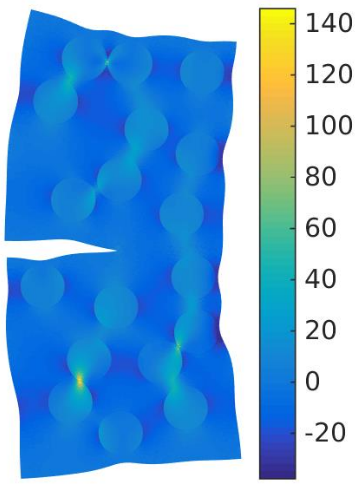



**Figure 5.** Stress field $\sigma_{yy}$ [MPa] at the saturated state corresponding to the maximum values of $L_{crack}$ and $c^{\infty}$.

Figure 6 illustrates the evolution of the stress $\sigma_{yy}(\mathbf{x}_1, t)$ and its response surface with repsect to $L_{crack}$ and $c^{\infty}$ for a particular time instant $t_1$ belonging to the transient part of the process. We notice that this part of the composite is in a compressive state. We can then observe important uncertainties on this quantity of interest mainly due to the randomness of the length of the crack since this material point can be close to the edge crack. Indeed, the response surface clearly points out that the length of the crack has a relevant influence unlike the maximum moisture content whose impact is especially noticeable at the end of the process.

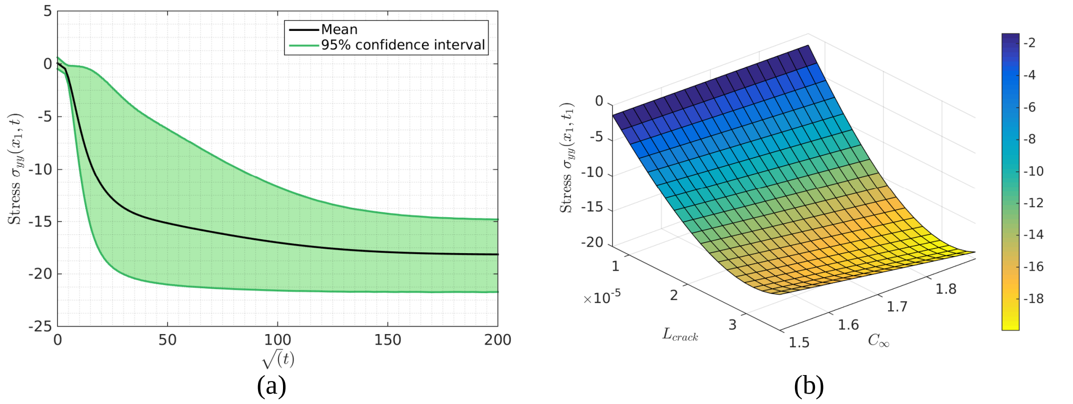


**Figure 6.** Evolution of the local stress $\sigma_{yy}(\mathbf{x}_1, t)$ [MPa] with respect to time (a) and response surface of $\sigma_{yy}(\mathbf{x}_1, t_1)$ with respect to $L_{crack}$ and $c^{\infty}$ (b) .

## 5. Conclusions

In this paper; the multiphysics modeling of the hygro-mechanical behavior of heterogeneous composite materials containing crack was investigated. We based our study on a classical Fick diffusion model associated to a linear elastic behavior taking into account the hygroscopic swelling. The deterministic problem is solved using the classical finite element method. We have addressed the input uncertainties which can lead to relevant randomness on the response of the physical model. We have proposed to tackle this difficulty using a parametric vision of the uncertainties and modeling the random local and global hygro-elastic response through polynomial chaos expansions. We finally showed the efficiency of the approach on a small study of a composite material containing an edge crack whose length was assumed random. The results have clearly exhibited relevant uncertainties on the response of the model for both diffusion and elastic quantities of interest.

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