Numerical stochastic study of damaged composite materials in humid environment

Alexandre Clément, Sylvain Fréour, Frédéric Jacquemin

GeM
University of Nantes / Ecole Centrale Nantes / CNRS UMR 6183

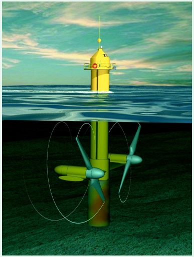
Work carried out within the framework of the WeAMEC, West Atlantic Marine Energy Community, and with funding from the CARENE and Pays de la Loire Region

Motivation

Model problem

Composite structures submitted to harsh environment

Tidal turbines



Offshore windmill

Complex coupled loadings

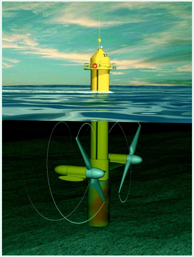
- Humidity
- Temperature
- Chemical aggressions
- Solar radiations
- Mechanical loadings

Motivation

Model problem

Composite structures submitted to harsh environment

Tidal turbines



Offshore windmill

Complex coupled loadings

- Humidity
- Temperature
- Chemical aggressions
- Solar radiations
- Mechanical loadings

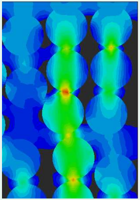
Challenges

1. Good understanding of the physical phenomena and their interactions
2. Development of an efficient and predictive multi-physics and multi-scale tool giving the structural response according to the material local behavior

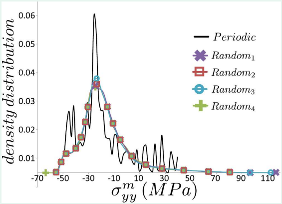
Interaction between moisture diffusion and mechanical behavior

Hygroscopic swelling

The moisture content leads to a so-called hygroscopic swelling involving relevant internal stresses [Peret et al. 2014]

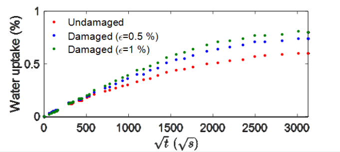


Local stress field

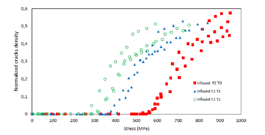


Stress level distribution within the composite

Moisture uptake and material damage



Water uptake for various damage levels

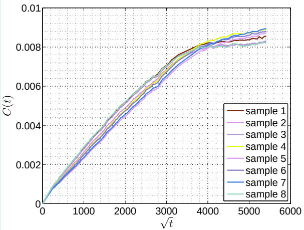


Crack density vs ageing [Tual 2015]

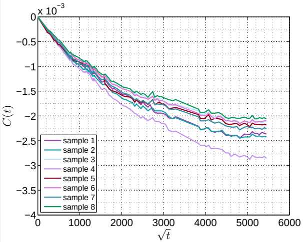
Variability observed for moisture diffusion problem

Typical experimental data of glass/polyamide composite material → uncertainties

Observations for glass/PA6-6 at HR= 80%



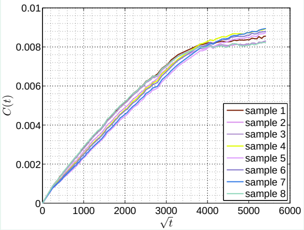
Observations for glass/PA6-6 at HR= 10%



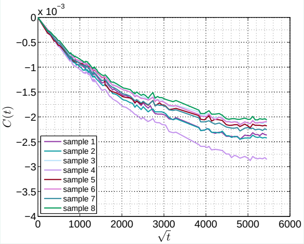
Variability observed for moisture diffusion problem

Typical experimental data of glass/polyamide composite material → uncertainties

Observations for glass/PA6-6 at HR= 80%



Observations for glass/PA6-6 at HR= 10%



Causes and different sources of uncertainties

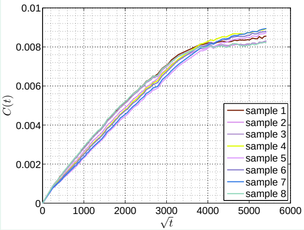
Causes of the observed variability

- Intrinsic variability of the material
- Measurement error
- Model error

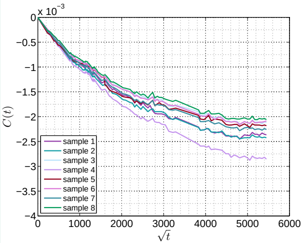
Variability observed for moisture diffusion problem

Typical experimental data of glass/polyamide composite material → uncertainties

Observations for glass/PA6-6 at HR$=80\%$



Observations for glass/PA6-6 at HR$=10\%$



Causes and different sources of uncertainties

Causes of the observed variability

- Intrinsic variability of the material
- Measurement error
- Model error

Input sources of uncertainties

- Random material parameters
- Random loadings
- Random geometries

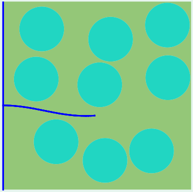
Outline

Outline

Hygro-mechanical problem with crack

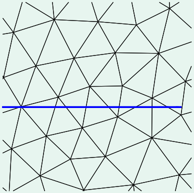
Objectives

- Study the effect of crack on the moisture diffusion
- Study the effect of moisture on the crack propagation
- Taking into account the various input uncertainties

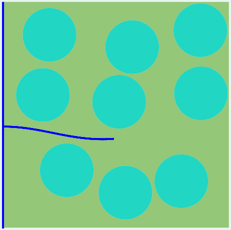


Why X-FEM modeling ?

- X-FEM methodology does not require conforming mesh
- X-FEM eases the crack propagation study
- X-FEM allows studying geometrical variability [Clément 2008]



Heterogeneous Fick problem: strong form



Find $c(\boldsymbol{x}, t) \in \Omega \times \mathbb{R}_*^+$ such that

$$\frac{\partial c(\boldsymbol{x}, t)}{\partial t} = \mathbf{D}\Delta c(\boldsymbol{x}, t) \quad \text{in} \quad \Omega \times \mathbb{R}_*^+$$

$$c(\boldsymbol{x}, t) = c^\infty \quad \text{on} \quad \Gamma_1 \times \mathbb{R}_*^+$$

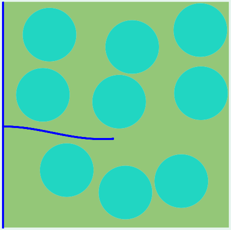
$$(\mathbf{D}\nabla_x c(\boldsymbol{x}, t)) \cdot \mathbf{n} = \mathbf{0} \quad \text{on} \quad \Gamma \backslash \Gamma_1 \times \mathbb{R}_*^+$$

$$c(\boldsymbol{x}, t = 0) = c_0(\boldsymbol{x}) \quad \forall \boldsymbol{x} \in \Omega$$

- where $\Omega = \Omega_1 \cup \Omega_2$ and $\mathbf{D} = \begin{cases} \mathbf{D}_1 \text{ if } \boldsymbol{x} \in \Omega_1 \\ \mathbf{0} \text{ if } \boldsymbol{x} \in \Omega_2 \end{cases}$
- the spatial average water content $C(t)$ verifies

$$C(t) = \frac{1}{M_0} \int_\Omega \rho(\boldsymbol{x})\, c(\boldsymbol{x}, t)\, d\Omega$$

Heterogeneous uncoupled hygro-elastic problem: strong form



Find $\boldsymbol{u}(\mathbf{x}, t) \in \Omega \times (0, T)$ such that

$$\begin{aligned} \boldsymbol{div}\,\boldsymbol{\sigma} + \boldsymbol{f} &= 0 && \text{on } \Omega\backslash\Gamma_{crack} \times (0, T) \\ \boldsymbol{\sigma} = \boldsymbol{C} : (\varepsilon^e(\boldsymbol{u}) &- \varepsilon^h(\boldsymbol{x}, t)) && \text{on } \Omega\backslash\Gamma_{crack} \times (0, T) \\ \boldsymbol{\sigma} \cdot \boldsymbol{n} &= \mathbf{0} && \text{on } \Gamma\backslash\Gamma_{crack} \times (0, T) \\ \boldsymbol{u} &= \boldsymbol{u}_{imp} && \text{on } \Gamma_u \times (0, T) \end{aligned}$$

- where $\Omega = \Omega_1 \cup \Omega_2$ and $\boldsymbol{C} = \begin{cases} \boldsymbol{C}_1 \text{ if } \boldsymbol{x} \in \Omega_1 \\ \boldsymbol{C}_2 \text{ if } \boldsymbol{x} \in \Omega_2 \end{cases}$
- with $\varepsilon^h(\boldsymbol{x}, t) = \begin{bmatrix} \beta^h_x c(\boldsymbol{x}, t) & 0 & 0 \\ & \beta^h_y c(\boldsymbol{x}, t) & 0 \\ \text{sym} & & \beta^h_z c(\boldsymbol{x}, t) \end{bmatrix}$

  $\rightarrow$ field $c(\boldsymbol{x}, t)$ can be obtained from any diffusion model

X-FEM methodology

eXtended Finite Element Method lies on two main aspects:

1. Implicit description of the geometry using the level-set technique [Sethian 1999]
2. Enriched approximation based on prior knowledge on the physical behavior [Moës et al. 1999]

Imposing Dirichlet BC with X-FEM for the diffusion problem

- Since cracks are not represent with a conforming mesh, imposing Dirichlet BC is not straightforward
- Use of the penalty approach [Fernandez et al. 2004] coupled to an enriched approximation to circumvent this issue → modified discretized system

$$(\mathbf{K} + \gamma \mathbf{K}^p)\mathbf{c} = \gamma \mathbf{f}^p \text{ where } \mathbf{K}^p = \int_{\Gamma_{crack}} N_i N_j d\Gamma \text{ and } \mathbf{f}^p = \int_{\Gamma_{crack}} N_i C_{imp} d\Gamma$$

$$c(\mathbf{x}) = \sum_i N_i(\mathbf{x}) c_i + \sum_i N_i(\mathbf{x}) H(\mathbf{x}) c_i^+ \qquad \text{with } H(\mathbf{x}) \text{ the Heaviside function}$$

Resolution of the elastic problem with hygroscopic strain

- Use of classical enrichment functions for the support and the tip of the crack [Moës et al. 1999]

Outline

A stochastic diffusion problem

A stochastic homogeneous Fick problem [Crank 1978]: strong form

$\rightarrow$ introduction of a finite dimensional probability space $(\boldsymbol{\Xi}, \mathcal{B}_{\boldsymbol{\Xi}}, P_{\boldsymbol{\Xi}})$ to model the different sources of uncertainties

Sources of uncertainties modeled with a finite set of random variables $\boldsymbol{\xi}$

- Random material parameters $\rightarrow \mathbf{D}(\boldsymbol{\xi})$
- Random loadings $\rightarrow c^{\infty}(\boldsymbol{\xi})$
- Random geometries $\rightarrow \Omega(\boldsymbol{\xi})$ and $\Gamma(\boldsymbol{\xi})$

Find $c(x, t, \boldsymbol{\xi}) : \Omega \times \mathbb{R}_*^+ \times \boldsymbol{\Xi} \rightarrow \mathbb{R}$ such that

$$
\begin{aligned}
\frac{\partial c(x,t,\boldsymbol{\xi})}{\partial t} &= \mathbf{D}(\boldsymbol{\xi}) \Delta c(x,t,\boldsymbol{\xi}) && \text{in} \quad \Omega(\boldsymbol{\xi}) \times \mathbb{R}_*^+ \times \boldsymbol{\Xi} \\
c(x,t,\boldsymbol{\xi}) &= c^{\infty}(\boldsymbol{\xi}) && \text{on} \quad \Gamma_1 \times \mathbb{R}_*^+ \times \boldsymbol{\Xi} \\
(\mathbf{D}(\boldsymbol{\xi}) \nabla_x c(x,t,\boldsymbol{\xi})) \cdot \mathbf{n} &= \mathbf{0} && \text{on} \quad \Gamma \backslash \Gamma_1 \times \mathbb{R}_*^+ \times \boldsymbol{\Xi} \\
c(x, t=0, \boldsymbol{\xi}) &= c_0(x, \boldsymbol{\xi}) && \forall x \in \Omega(\boldsymbol{\xi}) \times \boldsymbol{\Xi}
\end{aligned}
$$

A stochastic diffusion problem

A stochastic homogeneous Fick problem [Crank 1978]: strong form

→ introduction of a finite dimensional probability space $(\boldsymbol{\Xi}, \mathcal{B}_{\boldsymbol{\Xi}}, P_{\boldsymbol{\Xi}})$ to model the different sources of uncertainties

Sources of uncertainties modeled with a finite set of random variables $\boldsymbol{\xi}$

- Random material parameters → $\mathbf{D}(\boldsymbol{\xi})$
- Random loadings → $c^{\infty}(\boldsymbol{\xi})$
- Random geometries → $\Omega(\boldsymbol{\xi})$ and $\Gamma(\boldsymbol{\xi})$

Find $c(x, t, \boldsymbol{\xi}) : \Omega \times \mathbb{R}_*^+ \times \boldsymbol{\Xi} \to \mathbb{R}$ such that

$$
\begin{aligned}
\frac{\partial c(x,t,\boldsymbol{\xi})}{\partial t} &= \mathbf{D}(\boldsymbol{\xi}) \Delta c(x,t,\boldsymbol{\xi}) && \text{in} && \Omega(\boldsymbol{\xi}) \times \mathbb{R}_*^+ \times \boldsymbol{\Xi} \\
c(x,t,\boldsymbol{\xi}) &= c^{\infty}(\boldsymbol{\xi}) && \text{on} && \Gamma_1 \times \mathbb{R}_*^+ \times \boldsymbol{\Xi} \\
(\mathbf{D}(\boldsymbol{\xi}) \nabla_x c(x,t,\boldsymbol{\xi})) \cdot \mathbf{n} &= \mathbf{0} && \text{on} && \Gamma \backslash \Gamma_1 \times \mathbb{R}_*^+ \times \boldsymbol{\Xi} \\
c(x, t=0, \boldsymbol{\xi}) &= c_0(x, \boldsymbol{\xi}) && && \forall x \in \Omega(\boldsymbol{\xi}) \times \boldsymbol{\Xi}
\end{aligned}
$$

The stochastic modeling requires two steps:

1. the quantification of the uncertainties (*e.g.* the identification of the input random variables)
2. the propagation of uncertainties through a physical model leading to the characterization of the random response (*e.g.* probability density function of a quantity of interest, probability of failure, etc.)

Decomposition of the solution on a specific basis suited the stochastic problem:

The discrete solution $c(\boldsymbol{y}, \boldsymbol{\xi})$ will be searched under the form

$$c(\boldsymbol{y}, \boldsymbol{\xi}) \approx \sum_{\alpha=1}^{P} c_\alpha(\boldsymbol{y}) H_\alpha(\boldsymbol{\xi})$$

where the $c_\alpha(\boldsymbol{y})$ are the unknown of the problem and the $\{H_\alpha\}_{\alpha=1}^{P} \in L^2(\boldsymbol{\Xi}, dP_{\boldsymbol{\xi}})$ is a basis of orthonormal polynomials choosing with respect to the density of probability P_Ξ (Polynomial Chaos [Ghanem et al. 1991, Xiu 2002])

Decomposition of the solution on a specific basis suited the stochastic problem:

The discrete solution $c(\boldsymbol{y},\boldsymbol{\xi})$ will be searched under the form

$$c(\boldsymbol{y},\boldsymbol{\xi}) \approx \sum_{\alpha=1}^{P} c_\alpha(\boldsymbol{y}) H_\alpha(\boldsymbol{\xi})$$

where the $c_\alpha(\boldsymbol{y})$ are the unknown of the problem and the $\{H_\alpha\}_{\alpha=1}^{P} \in L^2(\boldsymbol{\Xi}, dP_{\boldsymbol{\xi}})$ is a basis of orthonormal polynomials choosing with respect to the density of probability P_{Ξ} (Polynomial Chaos [Ghanem et al. 1991, Xiu 2002])

L^2 projection method [Le Maître et al. 2010]

- Definition of approximation

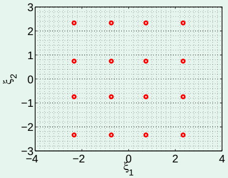
$$\mathbf{c}_\alpha = E(\mathbf{c}H_\alpha) = \int_{\Theta} H_\alpha(\boldsymbol{\xi})\mathbf{c}(\boldsymbol{\xi})dP_{\boldsymbol{\xi}}$$

- Computation with numerical integration

$\mathbf{c}_\alpha \approx \sum_{k=1}^{n_g} \omega_k H_\alpha(\boldsymbol{\xi}_k)\mathbf{c}(\boldsymbol{\xi}_k)$

where the $(\omega_k, \boldsymbol{\xi}_k)$ are the integration points

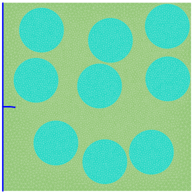
Example of quadrature with $n_g = 16$



$\rightarrow$ This technique only requires the resolution of n_g deterministic computations which can be made by a standard FEM software

Outline

Problem description

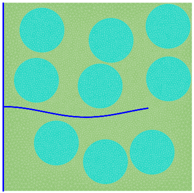


- Random crack length $L_{crack} \in U(5,\ 55)\,\mu m\,(cov = 50\%)$
- Random imposed $C^{\infty} \in U(1.6,\ 1.8)\ \%H20\,(cov = 4\%)$
- Isotropic moisture diffusion tensor with $D = 8.2\,10^{-2}\,\mu m^2/s$
- Volume fraction $v_f = 40\ \%$ with $d_f = 10\ \mu m$
- Epoxy elastic parameters : $E_m = 4\ GPa$, $\nu_m = 0.36$ et $\beta = 3.24\,\%$
- Glass elastic parameters : $E_f = 72.5\ GPa$, $\nu_m = 0.22$ et $\beta = 0\,\%$
- Vertical loading $\sigma_{22} = 1.5\ MPa$ on the top edge

Approximation parameters

- Spatial approximation with 14000 linear finite elements
- Euler's implicit time scheme for $T = 45\ h$ with $\Delta t = 10\,min$
- Penalty parameter $\gamma = 10^6$
- Stochastic approximation based on polynomial chaos with order $p = 3$

Problem description



- Random crack length $L_{crack} \in U(5,\ 55)\,\mu m\,(cov = 50\%)$
- Random imposed $C^{\infty} \in U(1.6,\ 1.8)\ \%H20\,(cov = 4\%)$
- Isotropic moisture diffusion tensor with $D = 8.2\,10^{-2}\,\mu m^2/s$
- Volume fraction $v_f = 40\ \%$ with $d_f = 10\ \mu m$
- Epoxy elastic parameters : $E_m = 4\ GPa$, $\nu_m = 0.36$ et $\beta = 3.24\,\%$
- Glass elastic parameters : $E_f = 72.5\ GPa$, $\nu_m = 0.22$ et $\beta = 0\,\%$
- Vertical loading $\sigma_{22} = 1.5\ MPa$ on the top edge

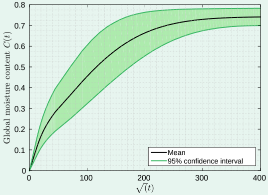
Approximation parameters

- Spatial approximation with 14000 linear finite elements
- Euler's implicit time scheme for $T = 45\ h$ with $\Delta t = 10\,min$
- Penalty parameter $\gamma = 10^6$
- Stochastic approximation based on polynomial chaos with order $p = 3$

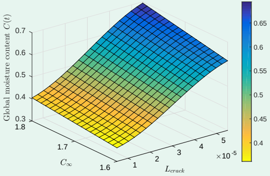
Stochastic numerical study: diffusion results

Results on the global moisture content $C(t)$

Random sorption curve

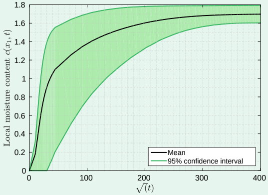


Response surface at $t = 4\ h$

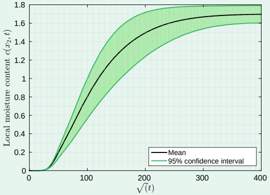


Results on the local moisture content $c(\mathbf{x}, t)$

$c(\mathbf{x_1}, t)$ with x_1 close to the crack

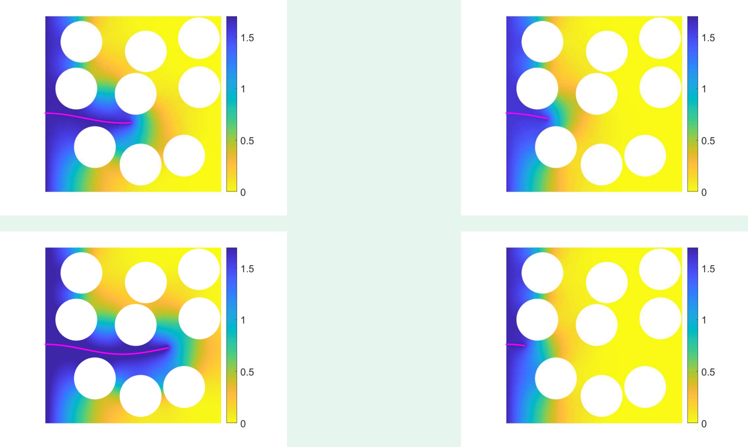


$c(\mathbf{x_2}, t)$ with x_2 further from the crack



$\rightarrow$ relevant influence of the crack length on both global and local moisture contents

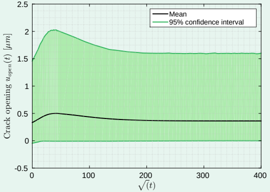
Random realizations of moisture field $c(\mathbf{x}, t)$ at $t = 0.5\ h$



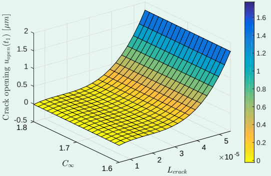
$\rightarrow$ fast post-processing of the complete stochastic solution

Results on the vertical crack opening $u_{open}(t)$

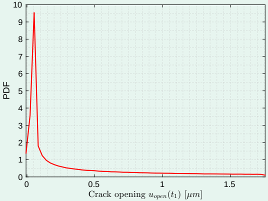
Stochastic process $u_{open}(t)$



Response surface at $t_1 = 35\ h$



PDF at $t_1 = 35\ h$



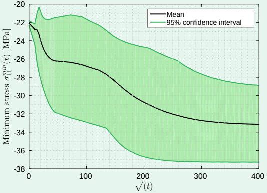
$P(u_{open}(t_1) > 1.5) = 4\%$

$\rightarrow$ significant variability mainly due to the crack length randomness

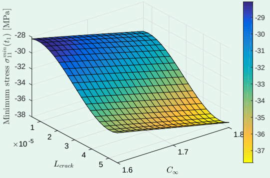
Stochastic numerical study: mechanical results

Results on the minimum local stress $\sigma_{11}^{min}(t)$

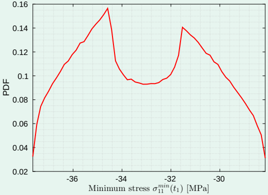
Stochastic process $\sigma_{11}^{min}(t)$



Response surface at $t_1 = 35\ h$



PDF at $t_1 = 35\ h$



$P(\sigma_{11}^{min}(t_1) \leqslant -37) = 4.7\%$

$\rightarrow$ significant variability mainly due to the crack length randomness but also to the maximum absorption capacity randomness

Conclusions and future works

Conclusions

- Study of the impact of hygroscopic aging on damage composites in a stochastic context
- Spectral stochastic approaches allow efficiently dealing with variability observed in hygro-mechanical problems
- X-FEM framework allows dealing with random geometrical diffusion problems (crack and others)

Ongoing and future works

- Dealing with the random crack propagation problem coupled with diffusion phenomenon
- Gathering experimental data for obtaining real input parameters and validating the numerical approach

Thank you for your attention

Selective bibliography I

T. Peret, A. Clement, S. Freour and F. Jacquemin.
Numerical transient hygro-elastic analyses of reinforced Fickian and non-Fickian polymers.
Composite Structures, doi:10.1016/j.compstruct.2014.05.026, 2014.

J. Crank.
The mathematics of diffusion.
Journal of Composite Materials, 12, 118-131, 1978.

N. Tual.
Durability of carbon/epoxy composites for tidal turbine blade applications.
Thèse de doctorat, Université de Bretagne Occidentale, 2015.

A. Clément
Elements finis stochastiques étendus pour le calcul de structures à géométrie aléatoire.
Thèse de doctorat, Université de Nnates, 2008.

J.A. Sethian.
Level Set Methods and Fast Marching Methods: Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision, and Materials Science.
Cambridge University Press, Cambridge, UK, 1999.

N. Moës, J. Dolbow, and T. Belytschko.
A finite element method for crack growth without remeshing.
Int. J. for Numerical Methods in Engineering, 46:131–150, 1999.

Selective bibliography II

S. Fernandez-Mendez and A. Huerta.
Imposing essential boundary conditions in mesh-free methods.
Computer Methods in Applied Mechanics and Engineering, 193, 1257-1275, 2004.

R. Ghanem and P. Spanos.
Stochastic finite elements: a spectral approach.
Springer, Berlin, 1991.

D. B. Xiu et G. E. Karniadakis.
The Wiener-Askey polynomial chaos for stochastic differential equations.
SIAM J. Sci. Comput., 24(2):619–644, 2002.

O. P. Le Maître and O. M. Knio.
Spectral Methods for Uncertainty Quantification with Applications to Computational Fluid Dynamics.
Springer, Heidelberg 2010.